

The Teaching and Learning of Algebra for At-Risk Students: Identifying the “Best Practices”

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“Algebra is the language of mathematics. It opens doors to more advanced mathematical topics for those who master basic algebraic concepts. It closes doors to college and to technology-based careers for those who do not. Those most seriously affected by lack of algebraic skills are students from minority groups.” (Lacampagne, 1995, p. 237)

Recent reform efforts across the nation have addressed the need for all students to learn algebra. All means ALL. Algebra is typically considered the “gatekeeper” for higher math courses, college, and many vocational careers. Therefore, algebra should not be restricted to the students with perceived high ability. Just as society has changed, the job market and the skills needed to function in society have changed. Therefore, how, when, why, and to whom we teach algebra MUST be changed. Educators can not expect to promote growth and prosperity in mathematics and society if algebra is only taught to the elite few. Our nation needs to maintain its standards of excellence through the twenty-first century. “The American future cannot rest on White males alone, for, by the

year 2000, only 15 percent of new workers will be White

males; the remaining 85 percent will be women, minorities, and immigrants” (Kamii, 1990, p.392).

The 1995 Virginia Standards of Learning stated that all students must complete a course in Algebra. Although the algebra curriculum has changed, “anyone familiar with the day to day realities of teaching students in a low achieving track knows that simply changing the curriculum is not sufficient” (Chazman, 1994, p. 9). The new curriculum and graduation requirement have many school administrators and teachers concerned with how to best meet the needs of at-risk students. The members of the School-University Research Network (SURN) are committed to facilitating the effort to educate all students in algebra. The first step in

helping teachers implement the new curriculum in a meaningful way must be to identify the “best practices” in teaching and learning algebra.

One recent reform effort included the “The Algebra Initiative Colloquium” that took place in December of 1993. The colloquium was a joint effort by the US Department of Education, the Office of Educational Research and Improvement, and the National Institute on Student Achievement, Curriculum, and Assessment. The papers presented at the Colloquium provided much insight on what needs to be done to insure algebra access for all students.

The National Council of Teachers of Mathematics (NCTM) developed a framework that emphasized the important decisions needed for effective mathematical teaching and learning to take place. The NCTM (1991) recommended that mathematics teachers:

- Set appropriate goals and select or create mathematical *tasks* that help students achieve these goals;
- Stimulate and manage classroom *discourse* so that both the students and the teacher are clearer about what is being learned;
- Create a positive classroom *environment* to support the teaching and learning of mathematics;
- Continually *analyze* student learning, the mathematical

tasks, and the environment in order to make ongoing instructional decisions.

This framework certainly can be applied to providing success in the algebra classroom. Thus, the “best practices” for teaching and learning algebra are discussed in conjunction with this framework.

Best Practice #1: Selecting and using meaningful algebraic tasks

In a study done by the Center for Research on Effective Schooling for Disadvantaged Students at Johns Hopkins University, Epstein and Mac Iver (1992) reported that students who are frequently taught math via problem solving situations (compared to students who are not frequently taught math with problem solving situations) have higher math achievement and are more motivated to ask questions to advance their understanding. There has certainly been a push to move algebra away from the emphasis on manipulation skills and toward conceptual understanding and problem solving (Booth, 1989). Thus, teachers need to provide students with meaningful tasks that develop critical thinking skills and enhance appreciation for the usefulness of algebra. These mathematical tasks should be of a genuine nature -- not the artificial contrived word problems found in many texts (Davis, 1989).

Algebra in the light of technology

Algebraic tasks should utilize appropriate technology whenever possible. Calculators (graphing and

scientific) and computers can give students opportunities to explore applications and concepts that would be too tedious and time consuming using paper-and-pencil techniques. Graphing tools have influenced the content of algebra by allowing:

- 1) ready visualization of relationships,
- 2) accurate solutions to equations and inequalities not possible by paper-and-pencil,
- 3) numerical and graphical support for paper-and-pencil solutions,
- 4) exploration in the effect of change on a representation,
- 5) exploration of concepts and relationships, and
- 6) modeling of real world phenomena (NCTM, 1995a).

Graphing technology can make algebra tasks exciting and motivating. Teachers need to take full advantage of any technology available to them and use it regularly in the classroom. Our world has changed in the light of technology...and so has algebra.

Algebra as the study of functions

Many documents (Harvey, 1991; Thorpe, 1989; NCTM, 1989; NCTM, 1995a) have supported the idea of using the concept of function as a central theme to algebra. Algebraic tasks that explore functional relationships can allow students to connect algebraic thought to everyday relationships that they are familiar and interested in (e.g.,

how many hours needed to work making x dollars per hour in order to save enough money to buy a stereo).

The functional approach to algebra is based on understanding quantities and quantitative relationships in varying contexts (Thompson & Thompson, 1995). From the very beginning, algebraic tasks are centered around patterns and relationships between variables. This approach to algebra can be rich with real world connections, deep conceptual understandings, and applications.

Best Practice #2: Stimulating classroom discourse

Meaningful classroom discourse can greatly influence the depth and breadth of the mathematics students learn. Students should be actively engaged in making conjectures, proposing approaches and solutions to problems, and validating their claims (NCTM, 1995a). A major goal of discourse should be for teachers to listen to the students and to act on what is being said in order to foster mathematical understanding (Burrill, 1995). In 1994, Chazman, an urban algebra teacher, firmly stated "I will be more effective with more students if I have students explain how they are thinking. The better they can articulate their understandings and the more I can understand their mathematical thinking, the more they will learn and the better I will be able to aid in this process" (p.7). The essence of effective classroom discourse can be summed up in three words -- *teacher as facilitator*.

Algebra for conceptual understanding

Classroom discourse will promote deeper conceptual understanding of algebraic ideas. Students should play an active role in constructing their own knowledge (Chazman, 1994). Teachers should not solely require students to perform rote procedures like “simplify”, “solve”, or “factor” without conceptual understanding of each task and why and when to utilize the learned skill. In order to teach conceptually, teachers must have their own conceptual orientation including:

- 1) a clear image of a concept and related concepts that students can develop,
- 2) a clear idea of how students can develop these concepts and ways of thinking,
- 3) ideas of which materials, activities, and expositions and students' engagement with them will foster conceptual development, and
- 4) an expectation that all students be intellectually engaged in tasks (Thompson & Thompson, 1995).

If conceptual understanding is so important, then exactly what algebraic understandings should students develop? Schoenfeld (1995) reported the following list as those that *all* students should learn:

- The representation of phenomena with symbols and the use of these symbols sensibly;
- The use of variables to describe patterns and give formulas involving geometric, physical, economic, and other relationships;
- Simple manipulations with these variables to enable other patterns to be seen and variations to be described;
- The solving of simple equations and inequalities and systems by hand and of more complicated equations and inequalities and systems by machine; and
- The picturing and examination of relationships among variables using graphs, spreadsheets or other technology. (p. 13)

This approach to algebra will certainly be different from the traditional “skill, drill and kill” method. However, students who are involved in appropriate classroom discourse will gain knowledge that is useful to them in the real world as well as prepare them for future mathematical encounters and higher level courses.

Best Practice #3: Creating a positive algebraic learning environment:

The classroom environment is largely dependent on the teacher. Success for all students in algebra must start with teachers believing that all students can and should learn algebra (Enneking, 1995). Teachers must commit themselves to teaching *all* students and be prepared to develop interesting and meaningful lessons that fully engage *all* students. The Algebra Project, originating in Boston, MA, based its theory and practice on the conviction that all students can learn algebra. One of the main components of the program was “an expectation of achievement that is reinforced by students, teachers, parents, and administrators” (Silva & Moses, 1990, p. 375). Furthermore, Silva and Moses, 1990, report that a broad goal of The Algebra Project was to produce teachers that can serve as facilitators in a learning environment based on real life experiences and supportive of the social construction of mathematics. There is evidence from effective mathematics programs (Steen, 1992) that students learn better in environments that utilize mathematics rich in context, community, and connections.

The NCTM (1989) encourages teachers to foster students’ mathematical power by respecting and valuing their ideas, ways of thinking and disposition. Many of these student characteristics are dependent upon gender, cultural background, home environment, and past mathematical experiences. Thus, sensitivity and awareness of students’ diverse backgrounds and experiences are essential in creating a positive learning environment for all students.

Recognizing and respecting cultural diversity

Many at-risk students in schools have diverse cultural backgrounds and often speak English as their second language (or even speak totally in their native tongue). Teachers need to recognize and respect student differences and adapt teaching and assessment methods to meet the needs of students’ diverse learning styles. Cuevas (1995) suggests four specific areas that can connect with the cultural diversity of students:

- study the historical roots of mathematics,
- use mathematics as a tool to study other cultures and countries,
- make students aware that others in their cultural group use mathematics in their careers, and
- establish strong home/school relations.

These four suggestions will give students of diverse backgrounds a personal connection in the mathematics studied as well as broaden the knowledge base of all students in the classroom.

Best Practice #4: Analyzing teaching and learning in algebra

Indeed, the best indicator of effective teaching is student learning. Teachers should constantly monitor student learning and make informed instructional decisions on what to do

next. Awareness of “what worked” and “what didn’t work” in a lesson needs to be fine tuned. Lesson plans can no longer be so rigid and time constrained that flexibility is an impossibility. At-risk students may require extra time and varying teaching strategies to achieve the necessary algebraic understanding. By continually using formative evaluation of students’ progress and analyzing the effectiveness of instructional techniques, teachers will empower themselves to help all students learn algebra. It can be done!

Teachers tend to teach the way they have been taught. Ongoing professional development is an integral key in maintaining a successful mathematical classroom. Teachers and administrators need to be committed to a strong professional development program that allows teachers to enhance their teaching skills and subject matter knowledge.

Varying assessment activities

“Although assessment is done for a variety of reasons, its main goal is to advance students’ learning and inform teachers as they make instructional decisions” (NCTM, 1995b, p. 13) Algebraic applications and conceptual understanding can not be accurately assessed with traditional multiple choice tests. In order to empower students to demonstrate their mathematical knowledge, multiple opportunities and various assessment strategies must be used (Cuevas, 1995). To help students increase their mathematical power, the NCTM (1995b) encouraged several shifts in assessment practices:

- toward assessing students’ progress in attaining mathematical power, and away from assessing students’ knowledge of facts and isolated skills,
- toward continually communicating with students about their performance in a comprehensive manner, and away from simply marking answers right or wrong,
- toward using multiple and complex assessment tools (i.e., performance tasks, projects, journals, oral demonstrations, and portfolios), and away from sole reliance on answers to brief questions on quizzes and chapter tests,
- toward students learning to assess their own progress, and away from teachers and external agencies (i.e., standardized tests) as the sole judges of mathematical progress.

Since all students can benefit from open-ended problems, group work, active discourse, and multidimensional learning (Steen, 1992), then all students can also benefit from assessment that truly evaluates their mathematical progress in those areas. Assessment is an invaluable and integral piece in the analysis of teaching and learning. Use it wisely.

Summary

Algebra has long been considered the gatekeeper to higher math and technology based careers. Now, algebraic thinking is considered a literacy requirement for citizenship -- along with reading and writing (Schoenfeld, 1995). If all students are to study algebra, then the way we teach algebra must be centered around conceptual understanding and problem solving. It is no longer necessary for every student to manipulate long complicated expressions and equations with speed and accuracy. Students must use these skills in context and see exactly how algebra is relevant to their lives.

The “best practices” discussed in this paper give algebra teachers a guideline for what is necessary to teach algebra to all students. By using meaningful algebraic tasks, maintaining classroom discourse, creating a positive learning environment, and continually analyzing teaching and learning, teachers will be engaging in some of the best practices for teaching algebra. The challenge is a big one -- but, with teachers, administrators, parents and students working together, “algebra for all” can become a reality.

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